

10 Partial Order Relation

65. Let \mathbb{N} be the set of natural numbers. On \mathbb{N} , we define a relation R by:

$$xRy \iff \exists z \in \mathbb{N} : xz = y.$$

Verify that R is a partial order relation.

66. Let \mathbb{R} be the set of real numbers. On \mathbb{R} , we define a relation R by:

$$aRb \iff \exists k \in \mathbb{N}_0 : b = 2^k a.$$

- (i) Write at least five elements of the relation R .
- (ii) Is R a partial order relation?
- (iii) Is R a linear order relation?

Provide detailed justifications for all answers.

67. Consider the divisibility relation $|$, defined on the set of positive integers, where $a|b$ is read as “ a divides b ”.

- (i) Show that $|$ is a partial order relation on \mathbb{Z}^+ .
- (ii) Show that $|$ is not a linear order relation on \mathbb{Z}^+ .

68. Let $S = \mathbb{Z}$ be the set of integers. On S , we define a relation R by:

$$aRb \iff \exists r \in \mathbb{N} : b = a^r.$$

- (a) Is R a partial order relation?
- (b) Is R a linear order relation?

69. Let \mathbb{Z} be the set of integers. On \mathbb{Z} , we define a relation R by:

$$xRy \iff \exists n \in \mathbb{N}_0 : x = y + n.$$

Is R a partial order relation? Is R a linear order relation? Provide detailed justifications for your answers.

70. Let B_4 be the set of natural numbers from 0 to 15. We represent these numbers in binary notation: the number $b \in B_4$ is written as $b = b_3b_2b_1b_0$, where each digit b_i is either 0 or 1 (specifically, $b = b_32^3 + b_22^2 + b_12^1 + b_02^0$, e.g., $8 = 1000$, $2 = 0010$, $15 = 1111$, etc.). On B_4 , we define a relation \preceq by:

$$a \preceq b \iff \forall i (a_i \leq b_i).$$

- (i) Write at least five elements of the relation \preceq .
- (ii) Is \preceq a partial order relation?
- (iii) Is \preceq a linear order relation?

Provide detailed justifications for all answers.

All above math problems are taken from the following website:

<https://osebje.famnit.upr.si/~penjic/teaching.html>.

THE READER CAN FIND ALL SOLUTIONS TO THE GIVEN PROBLEMS ON THE SAME PAGE.